L3 - Sciences Physiques et Chimiques. Electromagnétisme - Mai 2011.

I. $\mathrm{div}\,\vec{D} = \rho_{au} = \rho_\ell + \rho_f$; $\mathrm{div}\,\vec{B} = 0$; $\vec{\mathrm{rot}}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$; $\vec{\mathrm{rot}}\,\vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_{au} = \vec{J}_\ell$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$; $\vec{H} = (\vec{B}/\mu_0) - \vec{M}$.

II. 1) Le champ auxiliaire $\vec{E}^*_{in} = \frac{\rho_0 \vec{r}}{2\varepsilon_0} \Rightarrow V_m(\vec{r}) = \frac{\vec{P}}{\rho_0}.\vec{E}^* = \frac{\vec{P}.\vec{r}}{2\varepsilon_0} = 0, \forall \vec{r}$

($\vec{r}$ vecteur radial $\perp \vec{P}$) $\Rightarrow \vec{E}_m = -\vec{\mathrm{grad}}\,V_m = \vec{0}$

$\vec{E}_{in} = \vec{E}_a + \vec{E}_{m,in} = E_0 \sin\omega t\ \vec{e}_z \Rightarrow \vec{D}_{in} = \varepsilon_0 \cdot \vec{E}_{in} + \vec{P} = \varepsilon_0 \varepsilon_r \cdot \vec{E}_{in}$

2) $\vec{D}_{in} = \varepsilon_0 \varepsilon_r E_0 \sin\omega t . \vec{e}_z$; $\vec{\mathrm{rot}}\,\vec{H}_{in} = \frac{\partial \vec{D}_{in}}{\partial t} = \varepsilon_r \varepsilon_0 E_0 \omega \cos\omega t . \vec{e}_z$

$\vec{H}_{in}$ pseudovecteur $\Rightarrow \vec{H}_{in}$ selon $\vec{e}_\varphi$ et indépendant de φ (invariance) et de z (invariance) $\Rightarrow \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}(\rho . H_\varphi) . \vec{e}_z = \varepsilon_0 \varepsilon_r . E_0 \omega . \cos\omega t . \vec{e}_z$

$\Rightarrow H_\varphi = \frac{1}{2} . \rho\, \varepsilon_0 \varepsilon_r E_0 \omega \cos\omega t = \frac{B_{in,\varphi}}{\mu_r \mu_0} \Rightarrow B_{in,\varphi} = \frac{\rho}{2c^2} \varepsilon_r \mu_r E_0 \omega \cos\omega t$

III. 1. $\vec{\mathrm{rot}}(\vec{\mathrm{rot}}\,B) = \vec{\mathrm{rot}}(\mu_0 \vec{J}) = \mu_0\, \vec{\mathrm{rot}}\,\vec{J} = -\mu_0 \frac{n e^2}{m} \underbrace{\vec{\mathrm{rot}}\,A}_{=B} = \vec{\mathrm{grad}}(\underbrace{\mathrm{div}\vec{B}}_{0}) - \Delta \vec{B}$

$\Rightarrow \Delta \vec{B} = \frac{\vec{B}}{\lambda^2}$; $\lambda \simeq 1{,}7\ 10^{-8}$ m $= 17$ nm $\Leftrightarrow$ fine pellicule à la surface.

2. Invariance par translation en y et z $\Rightarrow \Delta \equiv \frac{\partial^2}{\partial x^2}$.

$\Rightarrow B(x) = \alpha \exp\frac{x}{\lambda} + \beta \exp -\frac{x}{\lambda}$ et $\alpha = 0$ car $B \to 0$ si $x \to \infty$.

3. $\vec{J} = \frac{1}{\mu_0} \vec{\mathrm{rot}}\,\vec{B} = \frac{1}{\mu_0} \begin{vmatrix} d/dx \\ 0 \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ 0 \\ B_0 \exp -\frac{x}{\lambda} \end{vmatrix} \Rightarrow \vec{J}(x) = \frac{B_0}{\mu_0 \lambda} e^{-x/\lambda} . \vec{e}_y$

4. $d^2\vec{f} = \vec{J}.d\tau \times \vec{B} = d\tau \begin{vmatrix} 0 \\ J_y \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ 0 \\ B_0 e^{-x/\lambda} \end{vmatrix} = \underbrace{dS\,dx}_{d\tau} \frac{B_0^2}{\mu_0 \lambda} e^{-\frac{2x}{\lambda}} \underbrace{(\vec{e}_y \times \vec{e}_z)}_{\vec{e}_x}$

$\Rightarrow \frac{d\vec{f}}{dS} = \int_0^\infty \frac{d^2\vec{f}}{dx\,dS} . dx = \frac{B_0^2}{\mu_0 \lambda} \int_0^\infty e^{-\frac{2x}{\lambda}} d\lambda = \frac{B_0^2}{2\mu_0} . \vec{e}_x$. C'est la pression magnétique, exprimée en N/m² et dirigée vers le milieu supra.

5. $\vec{J}_S = \int_0^\infty \vec{J}(x)\,dx = \int_0^\infty -\frac{1}{\mu_0} . \frac{dB}{dx}(x) . dx\ \vec{e}_y = \frac{B_0}{\mu_0} . \vec{e}_y$ conforme à la relation de passage $(\vec{B}_{ext} - \vec{B}_{in})_t = \mu_0 . J_S$

6. $[\lambda^2] = [m][c^2] \frac{[\varepsilon_0]}{[e^2]} [L^3] \equiv [\text{Energie}] \times \underbrace{\frac{[\varepsilon_0] \times [L]}{[e^2]}}_{[\text{énergie électrostatique}]^{-1}} \times L^2 \equiv L^2 \Rightarrow [\lambda] = [L]$

IV. 1. $\phi = \iint \vec{B}.d\vec{S} = B(\rho) \times 2\pi\rho h = \phi_0$ (conservation du flux) ; $\Rightarrow B(\rho) = \phi_0 / (2\pi\rho h)$

2. $P = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} \left(\frac{\phi_0}{2\pi\rho h}\right)^2$; 3. $\vec{F} = \int \vec{P}\,dS\,\vec{e}_z = \int_a^b \frac{1}{2\mu_0} \left(\frac{\phi_0}{2\pi\rho h}\right)^2 \times 2\pi\rho . d\rho\ \vec{e}_z$

$\vec{F} = \frac{\phi_0^2}{4\pi \mu_0 h^2} \ln\left(\frac{b}{a}\right) . \vec{e}_z$

4. $|m\vec{g}| = |\vec{F}| \Rightarrow h_0 = \phi_0 \left(\frac{\ln(b/a)}{4\pi \mu_0 m g}\right)^{1/2}$